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Persistent currents and critical magnetic field in planar dynamics of charged bosons

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Abstract

The aim of the present paper is the analysis from both quantum mechanics and thermodynamic points of view of the Hall-type behaviour of a relativistic charged scalar particle. Starting with the Euler–Lagrange equation, we obtain the solution and the Landau-type energy levels which exhibit a general dependence on the exterior electric and magnetic fields and on the particle momentum. For an ultra-relativistic particle, the characteristic function allows us to derive the so-called *persistent currents*, the state equation and the magnetization. In the last section, we add a self-interacting contribution to the Lagrangian and we get the critical magnetic induction values when the symmetry of the model is restored.

1. Introduction

Today, it has become clear that methods developed in high-energy physics can be employed successfully to explain the basic physical laws ruling the behaviour of macroscopic systems, since they provide a simple theoretical basis that can be verified experimentally to a high degree of precision. In that it concerns mesoscopic systems which have intermediate size (being small enough so that they still move in a coherent way, yet big enough for some measurable consequences), a broad area of research has developed by taking into account the fundamental implications of quantum mechanics in two-dimensional samples [1, 2]. Recently, it turned out that a deep understanding of quantum effects in mesoscopic physics is of major importance for the theory of quantum computers, in terms of the braid matrices of non-abelian fractional quantum Hall (FQH) anyons, since the encoded quantum information is inaccessible to local interactions and decoherence [3].

In the last 20 years, while dealing with the quantum Hall effect (integer or fractional) [4], a whole range of theories has been developed in order to explain this challenging phenomena. Experimental data on modulation-doped GaAs film [1] point out plateaus of the Hall resistance which cannot be explained unless one takes into consideration models beyond the Drude model, exhibiting quantum effects in the formation of Landau levels.

Because of the fermion-pair formation in superconducting states, a simple (but non-trivial) model to investigate their Hall-type behaviour, within the context of quantum field theory at finite temperature, is the model of the $U(1)$-minimally coupled, $SO(3,1)$-invariant dynamics, of a complex scalar field, aided by suitable Riemann-function regularization (techniques) in order to deal with the temperature-induced additional contributions [5, 6]. Therefore, employing a conveniently defined complex variable involving both the configuration and the momentum (sub)space, we have integrated the corresponding $U(1)$-gauge invariant Klein–Gordon equation, with an appropriate choice of the 4-potential such that, to emphasize the externally applied magnetic and electric fields, we obtain solutions for the possible quantum states and for their Landau-type (relativistically generalized) energy-quantization law. Since the latter exhibits an additional intricate dependence on the exterior electric field and the particle momentum, which is unaccounted for by the usual Landau levels (only proportional to Larmor precession), we have systematically followed its consequences on the Gibbs-like characteristic function, free energy and the so-called persistent currents, employing a generalized version of the Riemann-function technique which was used by [7], in studying the thermalized transverse magnetic (TM) modes of electromagnetic radiation in Einstein's universe. As a bonus to this endeavour, eventually adding a quartically self-interacting term to the Lagrangian, we obtain the state equation for the electron-paired gas formation as well as the corresponding magnetization. Together with the critical magnetic induction values, where the initial symmetry is restored, the bulk of these results is new (or, at least, of novel interest) for it has not been explicitly considered in most of the previously published works.

2. Hall-type evolution of charged scalar field

As in [6], we start with the $U(1)$-gauge invariant Lagrangian for the massive charged scalar field,

$$\mathcal{L} = \eta^{ij}\psi^*_{;i}\psi_{;j} + m_0^2\psi^*\psi + \tfrac{1}{4}F^{ij}F_{ij}, \tag{1}$$

where F_{ij} is the Maxwell tensor and ‘;’ stands for the $U(1)$-gauge covariant derivative,

$$\psi_{;i} = \psi_{,i} - \mathrm{i}qA_i\psi. \tag{2}$$

We employ Cartesian coordinates, $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - \mathrm{d}t^2$, and fix the gauge as: $A_x = A_z = 0$, $A_y = B_0x$ and $A_4 = E_0x$, where E_0 and B_0 are the orthogonal electric and magnetic fields. Consequently, the Euler–Lagrange equation is:

$$\eta^{ij}\psi_{,ij} - 2\mathrm{i}qB_0x\psi_{,y} + 2\mathrm{i}qE_0x\psi_{,t} - \left[m_0^2 + q^2x^2(B_0^2 - E_0^2)\right]\psi = 0, \tag{3}$$

and we introduce the parameters

$$\Omega = \frac{qB_0}{m_0^2}, \qquad \alpha = \frac{qE_0}{m_0^2}. \tag{4}$$

The standard variables separation

$$\psi = \chi(\xi)\,\mathrm{e}^{-\mathrm{i}p\eta}\mathrm{e}^{\mathrm{i}\omega t}, \tag{5}$$

in the Compton recalibration $\xi = m_0\,x$, $\eta = m_0\,y$, leads to the following differential equation:

$$\frac{\mathrm{d}^2\chi}{\mathrm{d}\xi^2} + \left[\frac{\omega^2}{m_0^2} - p^2 - 1 - 2\left(\Omega p + \frac{\alpha\omega}{m_0}\right)\xi - \left(\Omega^2 - \alpha^2\right)\xi^2\right]\chi = 0. \tag{6}$$

For the complex variable $z = \Omega\xi + \alpha\Omega^{-1} + p$ and within the semi-relativistic approximation, $\varepsilon - \alpha\xi + \Omega^2\xi^2/2 \ll 2$, with $\varepsilon = \omega - m_0$, the above equation turns into an oscillator-type equation,

$$\Omega^2 \frac{d^2\chi}{dz^2} + \left(2\frac{\varepsilon}{m_0} + 2\frac{\alpha p}{\Omega} + \frac{\alpha^2}{\Omega^2} - z^2\right)\chi = 0, \tag{7}$$

leading to the following dependence of the Landau-type energy levels on the exterior fields and on the particle momentum:

$$\frac{\varepsilon_n}{m_0} = \left(n + \frac{1}{2}\right)\Omega - \frac{\alpha^2}{2\Omega^2} - p\frac{\alpha}{\Omega}. \tag{8}$$

In the coming sections, we shall generalize the analysis developed in [6], by considering an ultra-relativistic particle in a strong magnetic field, for which the relation (8) turns into

$$\frac{\varepsilon_n}{m_0} = \left(n + \frac{1}{2}\right)\Omega - p\frac{\alpha}{\Omega}. \tag{9}$$

3. Persistent currents

In this section, we are going to generalize the analysis developed in [6] by considering in the characteristic function, G, the influence of the additional parameter a defined as in (12) which, instead of being equal to 1, as has been taken in [6], is now explicitly dependent on the particle transversal momentum, p, and on the exterior electric and magnetic fields encoded in α and Ω. Therefore, we introduce the temperature-dependence of the theory via the dimensionless parameter

$$\delta = \frac{\Omega}{4\pi T} = \frac{\Omega\beta}{4\pi}, \tag{10}$$

where β is the usual thermodynamic parameter $\beta = 1/T$. In order to deal with the whole range of temperatures and discuss the system behaviour at low temperatures, it is convenient to keep δ as being arbitrary and apply a similar formalism as the one developed in [7], for the analysis of the TM modes of the electromagnetic field in Einstein's universe.

Let us start with the energy spectrum (9), written as

$$\omega_n = \frac{\Omega}{2}\left[2n + 1 - 2\frac{p\alpha}{\Omega^2}\right] = \frac{\Omega}{2}(2n + a), \tag{11}$$

where we have introduced the notation

$$a \equiv 1 - 2p\frac{\alpha}{\Omega^2}. \tag{12}$$

In the characteristic function

$$G = \ln Z = \sum_{n=1}^{\infty} \ln\left[1 - \exp(-\beta\omega_n)\right] = \sum_{n=1}^{\infty} \ln\left[1 - \exp(-2\pi(2n + a)\delta)\right], \tag{13}$$

we expand the logarithm and, by taking the derivative with respect to δ, we get

$$\frac{\partial G}{\partial \delta} = 2\pi \sum_{n=1}^{\infty} (2n + a) \sum_{k=1}^{\infty} \exp[-2\pi(2n + a)k\delta]. \tag{14}$$

Using the complex integral representation

$$e^{-x} = \frac{1}{2\pi i}\int_C ds\, x^{-s}\Gamma(s), \tag{15}$$

with $x = 2\pi(2n+a)k\delta$, the relation (14) becomes:

$$\frac{\partial G}{\partial \delta} = \frac{1}{\mathrm{i}} \int_C \mathrm{d}s (2\pi\delta)^{-s} \Gamma(s) \zeta(s) \sum_n (2n+a)^{1-s}, \tag{16}$$

which can be expressed in terms of the Euler, Riemann and Riemann's generalized functions as

$$\frac{\partial G}{\partial \delta} = \frac{1}{\mathrm{i}} \int_C \mathrm{d}s (2\pi\delta)^{-s} \Gamma(s) \zeta(s) 2^{1-s} \zeta[s-1, 1+a/2]. \tag{17}$$

By applying the Residues theorem for the poles $s = 0, 1, 2$ and expanding $\zeta[1+\mathrm{i}s, 1+a/2]$ up to the third order in $(a-1)$, we have come to the expression

$$\frac{\partial G}{\partial \delta} = 2\pi \left[\frac{1}{12} + \frac{a}{4} + \frac{a^2}{8} \right] - \frac{1}{\delta} \frac{a+1}{2} + \frac{\pi}{24\delta^2} \left[1 - \left(\frac{\pi^2}{4} - 2 \right)(a-1) + \left(\frac{7}{4}\zeta[3] - 2 \right)(a-1)^2 \right]. \tag{18}$$

This result has been used in [6], in the particular case $a = 1$, for deriving the main thermodynamic quantities of the system and the bound for the ratio of entropy to energy.

As we have mentioned above, the more general case

$$a \equiv 1 - 2p \frac{\alpha}{\Omega^2}$$

leads to the new characteristic function, in the first-order approximation in $\alpha p/\Omega^2$,

$$G = \ln(4\pi) + \frac{11}{48}\frac{\Omega}{T} - \frac{\pi^2}{6}\frac{T}{\Omega} - \ln\frac{\Omega}{T} - \frac{\alpha p}{\Omega^2} \left[\ln(4\pi) + \frac{1}{2}\frac{\Omega}{T} - \frac{\pi^2}{3}\left(2 - \frac{\pi^2}{4}\right)\frac{T}{\Omega} - \ln\frac{\Omega}{T} \right], \tag{19}$$

which clearly exhibits a supplementary nontrivial overall dependence on the magnetic and electric fields, encoded in Ω and α, on the particle momentum p and on the temperature.

The above result allows us to compute essential mesoscopic quantities, such as for example the so-called *persistent currents*, which have been observed in mesoscopic rings with a length smaller than the coherence length, at very low temperatures where they have been measured by two-point-contact superconducting quantum interference device (SQUID) detectors [2]. The idea is to express the partition function and the free energy in terms of the Aharonov–Bohm (AB) flux, ϕ, as being $\mathcal{F}(T,\phi) = -k_B T \ln[Z(T,\phi)]$, and to define the persistent currents by

$$I(T,\phi) = -2\pi q \frac{\partial \mathcal{F}(T,\phi)}{\partial \phi}, \tag{20}$$

or, in our case, by

$$I(T,\Omega) = 2\pi \frac{q^2}{m_0^2} T \frac{\partial G}{\partial \Omega}. \tag{21}$$

Using (19), we get the following expression:

$$I = 2\pi \frac{q^2}{m_0^2} \left\{ \frac{11}{48} + \frac{\alpha p}{\Omega^2} \left[\frac{T}{\Omega}(1 + 2\ln(4\pi)) - \frac{T^2}{\Omega^2}\pi^2 \left(2 - \frac{\pi^2}{4}\right) - 2\frac{T}{\Omega}\ln\frac{\Omega}{T} + \frac{1}{2} \right] - \frac{T}{\Omega} + \frac{T^2}{\Omega^2}\frac{\pi^2}{6} \right\}. \tag{22}$$

Since the quantum Hall effect can be observed for high magnetic fields, low temperatures, low electric fields, low electron density, high mobility and extremely high-quality samples, one may notice that (22), for $T \to 0$, it becomes

$$I = 2\pi \frac{q^2}{m_0^2} \left[\frac{11}{48} + \frac{1}{2} \frac{\alpha p}{\Omega^2} \right]. \tag{23}$$

Finally, for strong magnetic fields and nonrelativistic particles, with $p \sim \alpha/\Omega \ll 1$, one ends up, in the low-temperature limit, with the value

$$I_0 = 2\pi \frac{q^2}{m_0^2} \frac{11}{48}. \tag{24}$$

Exact numbers usually indicate that the dynamics are dominated by symmetry and topology, as has been noticed in the topological quantization of gauge fields [8].

4. State equation and magnetization

Using the characteristic function (19), the energy of the system can be computed as

$$\begin{aligned} E &= -\frac{\Omega}{4\pi} \frac{\partial G}{\partial \delta} = T^2 \frac{\partial G}{\partial T} \\ &= \left[1 - \frac{\alpha p}{\Omega^2} \right] T - \frac{\pi^2}{6\Omega} \left[1 + 2\frac{\alpha p}{\Omega^2} \left(\frac{\pi^2}{4} - 2 \right) \right] T^2 - \frac{11}{48}\Omega + \frac{\alpha p}{2\Omega}, \end{aligned} \tag{25}$$

leading to the following heat capacity,

$$C = \frac{\partial E}{\partial T} = \left[1 - \frac{\alpha p}{\Omega^2} \right] - \frac{\pi^2}{3\Omega} \left[1 + 2\frac{\alpha p}{\Omega^2} \left(\frac{\pi^2}{4} - 2 \right) \right] T \tag{26}$$

whose expression is dependent not only on the ratio T/Ω, but also on the model parameter $\alpha p/\Omega^2$. In the text-book expression of the free energy, $F = U - TS$, where $U = VE(T)$ is the internal energy within the volume V, we replace the entropy by

$$S = -\left(\frac{\partial F}{\partial T} \right)_V \tag{27}$$

and get the following first-order differential equation:

$$T \left(\frac{\partial F}{\partial T} \right)_V - F = -VE. \tag{28}$$

The corresponding homogeneous equation:,

$$T \left(\frac{\partial F}{\partial T} \right)_V = F, \tag{29}$$

has the solution

$$F = K_V(T)T \tag{30}$$

so that (28) becomes

$$T^2 \frac{\mathrm{d}K_V}{\mathrm{d}T} = -VE, \tag{31}$$

yielding respectively

$$K_V = -V \int \frac{E(T)}{T^2} \,\mathrm{d}T \tag{32}$$

and

$$F = K_V T = -VT \int \frac{E(T)}{T^2} \mathrm{d}T. \tag{33}$$

By defining the pressure as

$$P = -\left(\frac{\partial F}{\partial V}\right)_V = T \int \frac{E(T)}{T^2} \mathrm{d}T, \tag{34}$$

we end up with the following state equation:

$$P = \left[1 - \frac{\alpha p}{\Omega^2}\right] T \ln T - \frac{\pi^2}{6}\left[1 + 2\frac{\alpha p}{\Omega^2}\left(\frac{\pi^2}{4} - 2\right)\right]\frac{T^2}{\Omega} + \frac{11}{48}\Omega - \frac{\alpha p}{2\Omega} \tag{35}$$

pointing out, for $T \to 0$, the corresponding pressure

$$P_0 = \frac{11}{48}\Omega - \frac{\alpha p}{2\Omega}, \tag{36}$$

which vanishes for the following $U(1)$-kinetic configuration

$$\frac{\alpha p}{\Omega^2} = \frac{11}{24}. \tag{37}$$

The derivative of the free energy $\mathcal{F} = -T \ln Z = -TG$, with G given by (19), with respect to the magnetic induction, leads to the magnetization, whose dependence on the temperature and the external field intensities is given by:

$$M = -\frac{\partial F}{\partial B} = -\frac{q}{m_0^2}\frac{\partial F}{\partial \Omega} = \frac{q}{m_0^2}\left\{\frac{11}{48} - \frac{T}{\Omega} + \frac{T^2}{\Omega^2}\frac{\pi^2}{6} + \frac{\alpha p}{\Omega^2}\left[\frac{T}{\Omega}(1 + 2\ln(4\pi)) - \frac{T^2}{\Omega^2}\pi^2\left(2 - \frac{\pi^2}{4}\right) - 2\frac{T}{\Omega}\ln\frac{\Omega}{T} + \frac{1}{2}\right]\right\}. \tag{38}$$

One may notice that, in the particular case $\alpha p \approx 0$, corresponding to $a = 1$, the above relation becomes simplified, while the corresponding susceptibility (defined as the first derivative of M with respect to B), has the explicit expression

$$\chi = \frac{T}{B^2}\left[1 - \frac{m_0^2}{q}\frac{\pi^2}{3}\frac{T}{B}\right], \tag{39}$$

which vanishes for the critical value of temperature equal to $3/\pi^2 \times$ the ratio between the Planck–Larmor quanta and the rest energy.

5. The critical magnetic field

The novel bulk of concepts developed in particle physics, such as for example the spontaneously broken gauge symmetry and macroscopic occupation of quantum levels, have been recognized as key factors for understanding superconductivity and for explaining the Meissner effect or the flux quantization in Josephson and integer quantum Hall effects [9]. Even the simplest models used in domain walls, gauge strings, magnetic monopoles and textures, are based on Lagrangians of real or complex scalar fields, with a spontaneously broken gauge symmetry [10]. For example, the Nielson–Olesen model, based on a $U(1)$-gauge invariant Lagrangian for the complex scalar field with an additional self-interacting term leads, besides the infinite discrete set of soliton solutions, to quantization of the magnetic flux and of the Hall resistance [11].

Therefore, we change the sign in front of the mass term, replace m_0 by the mass parameter μ_0, and add a self-interacting contribution to the Lagrangian (1), so that the whole potential turns into the one with broken symmetry,

$$V(\psi,\psi^*) = -\mu_0^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4, \tag{40}$$

which, in the case of a real field ϕ, is perfectly compatible, because of the 1/2-rule (when switching from complex to real fields), with the well-known quartic potential

$$V(\phi) = -\frac{\mu_0^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4.$$

Further on, considering the Landau-type energy levels (11) as being given by

$$\omega_n = n\Omega + q, \tag{41}$$

we have to use path integral formalism. To first order in λ, the generating functional (per unit volume) reads

$$\ln W = -3\lambda \sum_n \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{\omega_n^2 + k^2 + \mu_0^2}\right]^2. \tag{42}$$

Thus, we have to add algebraically to the usual Feynman–Cooper propagator

$$\langle \psi(\vec{x}_1)\psi^*(\vec{x}_2)\rangle^{\Omega}_{\lambda=0} = C(\vec{x}_1 - \vec{x}_2), \tag{43}$$

where the bilocally generalized function C is the *Cooperon*, defined as the solution to the distribution-like differential equation

$$\left[\left(\nabla - iq\vec{A}(\vec{x})\right)^2 - \frac{1}{D\tau}\right] C(\vec{x}-\vec{y}) = -\frac{1}{D\tau}\delta(\vec{x}-\vec{y}),$$

with D being the diffusion coefficient and τ the mean lifetime of the Landau energy levels, the two-point function $-\mathcal{F}_\lambda(\vec{x}_1 - \vec{x}_2)$, i.e.

$$\langle 0|T\left[\psi(x_1)\psi^*(x_2)\right]|0\rangle^{\Omega}_{\lambda} = C(x_1 - x_2) - \mathcal{F}_\lambda(x_1 - x_2),$$

whose integral expression over momentum space reads

$$\mathcal{F} = 12\lambda\frac{\Omega}{2\pi}\sum_n \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_n^2 + k^2 + m_0^2}, \tag{44}$$

being nothing other than the self-energy due to the quartic interaction.

Switching to spherical coordinates in isotropic momentum space, i.e. $\int_{(4\pi)} d^3k = 4\pi k^2 dk$, and systematically isolating the divergent parts of $\mathcal{F}_\lambda$, due to the sums over the Landau quantum number $n \in N \cup \{0\}$, one ends up with the regularized self-energy expression

$$\underline{\mathcal{F}}_\lambda = -\frac{3\lambda}{2\pi^2}\mu_0^2\Omega\sum_{n=0}^{\infty}\left[\sqrt{\omega_n^2 + \mu_0^2} + \omega_n\right]^{-1}, \tag{45}$$

which can be written, because of the following row of relations

$$\sum_{n=0}^{\infty}\left(\sqrt{\omega_n^2+\mu_0^2}+\omega_n\right)^{-1} \approx \frac{1}{2}\sum_{n=0}^{\infty}\left[\omega_n\left(1+\frac{(\mu_0/2)^2}{\omega_n^2}\right)\right]^{-1}$$
$$\approx \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{\omega_n} - \frac{\mu_0^2/4}{\omega_n^3}\right] \approx -\frac{\mu_0^2}{8}\sum_{n=0}^{\infty}(n\Omega+q)^{-3} = -\frac{\mu_0^2}{8\Omega^3}\zeta\left(3,\frac{q}{\Omega}\right), \tag{46}$$

in the concrete form

$$\mathcal{F}_\lambda = \frac{3\lambda\mu_0^4}{16\pi^2\Omega^2}\zeta\left(3,\frac{q}{\Omega}\right). \tag{47}$$

This is certainly dependent not only on Ω, as it should be, but also on the particle's transversal momentum and the external electric field intensity encoded in the adjusted q-parameter, which is defined by the relation

$$q = \frac{\Omega}{2} - \frac{p\alpha}{\Omega^2}. \tag{48}$$

Thence, taking into account the one-loop corrections, one can define the effective potential, $\mathcal{V}$, whose second-order derivative is:

$$\mu^2 = \left.\frac{\partial^2 \mathcal{V}}{\partial \psi^2}\right|_{\psi=0} = -\mu_0^2 + \mathcal{F}. \tag{49}$$

The condition $\mu^2 = 0$ leads to the critical magnetic field value(s)

$$\Omega_c^2 = \frac{3\lambda\mu_0^2}{16\pi^2}\zeta\left(3, \frac{q}{\Omega_c}\right), \tag{50}$$

where the symmetry is restored.

For the case of non-relativistic particles in strong magnetic field, where

$$q = \Omega/2, \tag{51}$$

the critical value becomes

$$\Omega_c^2 = \frac{21\lambda\mu_0^2}{16\pi^2}\zeta(3), \tag{52}$$

and depends, besides the model parameters μ_0 and λ, on the transcendent quantity

$$\lambda^{-1}\left(\frac{\Omega_c}{\mu_0}\right)^2 \approx \frac{21\pi}{16}\frac{1}{25.794\,36\ldots}$$

6. Concluding remarks

The present paper deals with $U(1)$-gauge invariant analysis of the Hall-type evolution of charged scalar field. In Cartesian coordinates, the Klein–Gordon equation leads to Landau-type energy levels, exhibiting a general dependence on the exterior electric and magnetic fields and on the particle momentum. By introducing the thermodynamic dimensionless parameter δ, one is able to derive the characteristic function, in terms of the Aharonov–Bohm (AB) flux, and compute the *persistent currents*. Next, we obtained the state equation, expressing the pressure dependence on the temperature, particle momentum and external fields. The derived free-energy expression has allowed us to compute the magnetization and the corresponding susceptibility, pointing out the critical value of the temperature where the latter is vanishing. Finally, within the frame of path integral formalism, for the spontaneously broken $U(1)$-gauge symmetry, dealing with the regularized self-energy expression, we obtained the critical magnetic field irrational values for symmetry restoration. Last, but not least, because of the combined geometro-functional invariance of the employed formalism, we are pretty very confident that a similar—fully $SO(3,1) \times U(1)$-gauge invariant—approach would hold for more interesting astrophysical structures, such as the charged radiating rings of high-energy particles surrounding the poles of quantum degenerate highly magnetized stars.

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References

[1] Datta S 2003 *Electronic Transport in Mesoscopic Systems* (Cambridge: Cambridge University Press)

[2] Georgiev L S 2005 *Nucl. Phys.* B **707** 347
Georgiev L S 2004 *Phys. Rev.* B **69** 085305
Georgiev L S 2006 *Preprint* hep-th/0601075

[3] Sarma S D, Freedman M and Nayak C 2005 *Phys. Rev. Lett.* **94** 166802

[4] Tsui D C, Stömer H L and Gossard A C 1982 *Phys. Rev. Lett.* **48** 1559
Laughlin R B 1983 *Phys. Rev. Lett.* **50** 1395
Halperin B I 1984 *Phys. Rev. Lett.* **52** 1583

[5] Dariescu M A and Dariescu C 2005 *Chaos Solitons Fractals* **23** 1005
Dariescu M A, Dariescu C and Murariu G 2006 *Chaos Solitons Fractals* **28** 1

[6] Dariescu M A and Dariescu C 2007 *Chaos Solitons Fractals* **33** 776

[7] Brevik I, Herikstad R and Skriudalen S 2005 *Preprint* hep-th/0508123 and the references therein

[8] Cappelli A, Trugenberger C A and Zemba G 1993 *Nucl. Phys.* B **396** 465

[9] Fubini S and Molinari A 1993 *Ann. Phys.* **221** 1

[10] Vachaspati T 1993 Topological defects in cosmology *1993 Summer School in High Energy Physics and Cosmology* (*The ICTP Series in Theoretical Physics* vol 10) (Singapore: World Scientific) pp 137–74

[11] Dariescu M A and Dariescu C 2002 *Chaos Solitons Fractals* **13** 1851